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BEYOND KNUTH'S NOTATION FOR UNIMAGINABLE NUMBERS WITHIN COMPUTATIONAL NUMBER THEORY

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Abstract. Literature considers under the name "unimaginable numbers" any positive integer going beyond any physical application. One of the most known methodologies to conceive such numbers is using *hyper-operations*, that is a sequence of binary functions defined recursively starting from the usual chain: addition - multiplication - exponentiation. The most important notations to represent such hyper-operations have been considered by Knuth, Goodstein, Ackermann and Conway as described in this work's introduction. Within this work we will give an axiomatic setup for this topic, and then try to find on one hand other ways to represent unimaginable numbers, as well as on the other hand applications to computer science, where the algorithmic nature of representations and the increased computation capabilities of computers give the perfect field to develop further the topic, exploring some possibilities to effectively operate with such big numbers. In particular, we will give some axioms and generalizations for the up-arrow notation and, considering a representation via rooted trees of the hereditary base-n notation, we will determine in some cases an effective bound related to "Goodstein sequences" using Knuths notation. Finally, we will also analyze some methods to compare big numbers, proving specifically a theorem about approximation using scientific notation and a theorem on hyperoperation bounds for Steinhaus-Moser notation.

Keywords: Computational number theory, unimaginable numbers, Knuth's up-arrow notation, big data, number representation, Goodstein's Theorem

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