

## A NOTE ON THE STRUCTURE OF $\frac{\mathbb{F}_q S_5}{J(\mathbb{F}_q S_5)}$

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**ABSTRACT.** A complete algebraic structure of  $\frac{\mathbb{F}_q S_5}{J(\mathbb{F}_q S_5)}$ , where  $\mathbb{F}_q = \mathbb{F}_{p^n}$  is a finite field with characteristic  $p$  (a prime) and  $J(\mathbb{F}_q S_5)$  is the Jacobson radical of the group algebra  $\mathbb{F}_q S_5$  is given.

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### 1. Introduction

The problem of obtaining the structure of unit group of rings, in general, and group rings, in particular, is a difficult as well as an important one that has drawn attention of several researchers and a lot of work has been done in this direction (see [1], [2], [7]-[18], [23], [24]). The research in this area has gone beyond simply obtaining the structure of unit groups and efforts have also been made to investigate applications of units of group algebras. Dholakia [4] and Hurley [9] gave a method to construct convolutional codes by using units of group rings. Due to the complexity of the problem in the case of group algebras and group rings as well as their importance researchers have focussed on obtaining the structure of unit groups for specific fields and groups with special attention given to the symmetric groups and alternating groups.

In 2007, Sharma *et al.* determined the unit group of the group algebra  $\mathbb{F} S_3$  of symmetric group  $S_3$  and also of the group algebra  $\mathbb{F} A_4$  of alternating group  $A_4$  over any finite field  $\mathbb{F}$  (see [21], [22]). In 2008, Khan *et al.* [12] extended the result to the symmetric group  $S_4$ . In 2010, Gildea [6] determined the unit group of the group algebra  $\mathbb{F}_{2^k} A_4$ . Makhijani *et al.* [19] considered the case of the alternating group  $A_5$ .

Kumar *et al.* [13] considered the problem of obtaining the structure in the case of symmetric group  $S_5$  and gave the possible structure of the unit group of  $\mathbb{F}_{p^n} S_5$ , when  $p$  is a prime  $> 5$ . In this article, we continue this discussion of the unit group of the group algebra  $\mathbb{F}_{p^n} S_5$  of the symmetric group  $S_5$ .

If  $\mathbb{F}$  is a perfect field, then by Wedderburn-Malcev Theorem [3, Chapter 10, Theorem 72.19], it is well known that

$$\mathcal{U}(\mathbb{F}G) \cong (1 + J(\mathbb{F}G)) \rtimes \mathcal{U}\left(\frac{\mathbb{F}G}{J(\mathbb{F}G)}\right)$$

where  $J(\mathbb{F}G)$  denotes the Jacobson radical of the group algebra  $\mathbb{F}G$  and  $\mathcal{U}(A)$  denotes the unit group of the  $\mathbb{F}$ -algebra  $A$ . A Wedderburn decomposition of  $\frac{\mathbb{F}G}{J(\mathbb{F}G)}$  can, therefore, be very helpful to determine unit group of  $\mathbb{F}G$ . We, in this article, give the Wedderburn decomposition of  $\frac{\mathbb{F}S_5}{J(\mathbb{F}S_5)}$  for any finite field  $\mathbb{F}$  (see Theorem 2.13).

Throughout  $\mathbb{F}_q$  will denote the finite field  $\mathbb{F}_{p^n}$ , where  $p$  is a prime. We shall use the presentation

$$\langle a, b \mid a^2, b^5, (ab)^4, (bab^{-2}ab)^2 \rangle$$

where  $a = (1, 2)$  and  $b = (1, 2, 3, 4, 5)$  of  $S_5$ . The conjugacy classes of  $S_5$  are, then, given by the representatives  $e, a, b, ab, (ab)^2, ab^2$ , and  $abab^{-1}$ . Note that the orders of these representatives are 1, 2, 5, 4, 2, 6, and 3, respectively.

For a  $p$ -regular element  $g$  of a group  $G$ , we denote by  $\gamma_g$ , the sum of all conjugates of  $g$  in  $G$  and  $S_{\mathbb{F}}(\gamma_g)$ , the cyclotomic  $\mathbb{F}$ -class of  $\gamma_g$ , that is,

$$S_{\mathbb{F}}(\gamma_g) = \{\gamma_{g^t} \mid t \in T_{G, \mathbb{F}}\}.$$

We recall that the number of simple components of  $\frac{\mathbb{F}G}{J(\mathbb{F}G)}$  is equal to the number of cyclotomic  $\mathbb{F}$ -classes in  $G$  [5, Proposition 1.2]. For more details, we refer the reader to Ferraz [5]. Throughout the paper,  $M^{\beta}$  denotes the vector space spanned by  $\beta$ -tabloids,  $S^{\beta}$  represents the Specht module associated with the partition  $\beta$ , and  $D^{\beta} = \frac{S^{\beta}}{S^{\beta} \cap S^{\beta\perp}}$ . For more details about the Specht modules, we refer the reader to Chapter 4 of [10].

## 2. The structure of $\frac{\mathbb{F}_q S_5}{J(\mathbb{F}_q S_5)}$

**Lemma 2.1.** *If  $p = 5$ , then  $\dim_{\mathbb{F}_q} J(\mathbb{F}_q A_5) = 25$ .*

**Proof.** By [19, Theorem 3.6],  $\frac{\mathbb{F}_q A_5}{J(\mathbb{F}_q A_5)} \cong \mathbb{F}_q \oplus \mathbb{M}_3(\mathbb{F}_q) \oplus \mathbb{M}_5(\mathbb{F}_q)$ . The result now follows by dimension constraints.  $\square$

**Lemma 2.2.** *If  $p = 5$ , then  $\dim_{\mathbb{F}_q} J(\mathbb{F}_q S_5) = 50$ .*

**Proof.** The group  $G = S_5$  has a normal subgroup, namely  $A_5$ , of index 2, and since  $5 \nmid 2$ , by [11, Proposition 1.8, p.108], we have

$$\dim_{\mathbb{F}_q} J(\mathbb{F}_q S_5) = |S_5 : A_5| \cdot \dim_{\mathbb{F}_q} J(\mathbb{F}_q A_5) = 2 \cdot 25 = 50,$$

as desired.  $\square$

**Theorem 2.3.** *If  $p = 5$ , then*

$$\frac{\mathbb{F}_q S_5}{J(\mathbb{F}_q S_5)} \cong \mathbb{F}_q \oplus \mathbb{F}_q \oplus \mathbb{M}_3(\mathbb{F}_q) \oplus \mathbb{M}_3(\mathbb{F}_q) \oplus \mathbb{M}_5(\mathbb{F}_q) \oplus \mathbb{M}_5(\mathbb{F}_q).$$

**Proof.** Observe that an element  $x \in S_5$  is 5-regular if  $5 \nmid o(x)$ . We note that except for elements corresponding to the conjugate class of  $b$  are 5-regular. Now observe that  $5^n \equiv 1 \text{ or } 5 \pmod{12}$ . Hence  $S_{\mathbb{F}_q}(\gamma_g) = \{\gamma_g\}$  for each 5-regular element  $g \in S_5$ . Also as  $\dim_{\mathbb{F}_q} J(\mathbb{F}_q S_5) = 50$ ,  $\dim_{\mathbb{F}_q} \frac{\mathbb{F}_q S_5}{J(\mathbb{F}_q S_5)} = 70$ . Therefore  $\frac{\mathbb{F}_q S_5}{J(\mathbb{F}_q S_5)}$  has 6 simple components with total dimension 70, that is,

$$\frac{\mathbb{F}_q S_5}{J(\mathbb{F}_q S_5)} \cong \mathbb{F}_q \oplus \mathbb{M}_{n_1}(\mathbb{F}_q) \oplus \mathbb{M}_{n_2}(\mathbb{F}_q) \oplus \mathbb{M}_{n_3}(\mathbb{F}_q) \oplus \mathbb{M}_{n_4}(\mathbb{F}_q) \oplus \mathbb{M}_{n_5}(\mathbb{F}_q). \quad (1)$$

Using dimension constraints, we see from (1)

$$70 = 1 + n_1^2 + n_2^2 + n_3^2 + n_4^2 + n_5^2. \quad (2)$$

Now observe that assignments  $a \mapsto 1, b \mapsto 1$  and  $a \mapsto 1, b \mapsto -1$  give two irreducible  $\mathbb{F}_q$ -representations of  $S_5$  and we know that dimensions of Specht modules corresponding to conjugate partitions are same, that is,  $\dim_{\mathbb{F}_q} D^{(3,2)} = \dim_{\mathbb{F}_q} D^{(2,2,1)}$  and  $\dim_{\mathbb{F}_q} D^{(4,1)} = \dim_{\mathbb{F}_q} D^{(2,1,1,1)}$ . Hence the only possible solution of (2) satisfying these conditions is 1, 3, 3, 5, 5. Thus,

$$\frac{\mathbb{F}_q S_5}{J(\mathbb{F}_q S_5)} \cong \mathbb{F}_q \oplus \mathbb{F}_q \oplus \mathbb{M}_3(\mathbb{F}_q) \oplus \mathbb{M}_3(\mathbb{F}_q) \oplus \mathbb{M}_5(\mathbb{F}_q) \oplus \mathbb{M}_5(\mathbb{F}_q).$$

This proves the theorem.  $\square$

**Corollary 2.4.** *If  $p = 5$ , then*

$$\mathcal{U}(\mathbb{F}_q S_5) \cong (1 + J(\mathbb{F}_q S_5)) \rtimes (\mathbb{F}_q^* \times \mathbb{F}_q^* \times GL_3(\mathbb{F}_q) \times GL_3(\mathbb{F}_q) \times GL_5(\mathbb{F}_q) \times GL_5(\mathbb{F}_q)),$$

where  $1 + J(\mathbb{F}_q S_5)$  is a nonabelian subgroup of  $\mathcal{U}(\mathbb{F}_q S_5)$  with order  $5^{50n}$  and  $J(\mathbb{F}_q S_5)$  is a nilpotent ideal.

**Proof.** By Theorem 2.3, we have

$$\mathcal{U}\left(\frac{\mathbb{F}_q S_5}{J(\mathbb{F}_q S_5)}\right) \cong \mathbb{F}_q^* \times \mathbb{F}_q^* \times GL_3(\mathbb{F}_q) \times GL_3(\mathbb{F}_q) \times GL_5(\mathbb{F}_q) \times GL_5(\mathbb{F}_q).$$

Since  $\mathbb{F}_q$  is perfect,

$$\mathcal{U}(\mathbb{F}_q S_5) \cong (1 + J(\mathbb{F}_q S_5)) \rtimes (\mathbb{F}_q^* \times \mathbb{F}_q^* \times GL_3(\mathbb{F}_q) \times GL_3(\mathbb{F}_q) \times GL_5(\mathbb{F}_q) \times GL_5(\mathbb{F}_q)).$$

The non-abelian part follows from [11, Theorem 15.2, p.260]. Now  $|1 + J(\mathbb{F}_q S_5)| = 5^{50n}$ , because  $\dim_{\mathbb{F}_q} J(\mathbb{F}_q S_5) = 50$ .  $J(\mathbb{F}_q S_5)$  is nilpotent as  $S_5$  is finite. This completes the proof.  $\square$

**Lemma 2.5.** *If  $p = 3$ , then  $\dim_{\mathbb{F}_q} J(\mathbb{F}_q A_5) = 25$ .*

**Proof.** By [19, Theorem 3.6],

$$\frac{\mathbb{F}_q A_5}{J(\mathbb{F}_q A_5)} \cong \begin{cases} \mathbb{F}_q \oplus \mathbb{M}_3(\mathbb{F}_{q^2}) \oplus \mathbb{M}_4(\mathbb{F}_q), & \text{if } n \text{ is odd,} \\ \mathbb{F}_q \oplus \mathbb{M}_3(\mathbb{F}_q) \oplus \mathbb{M}_3(\mathbb{F}_q) \oplus \mathbb{M}_4(\mathbb{F}_q), & \text{if } n \text{ is even.} \end{cases}$$

The result now follows by using dimension constraints.  $\square$

**Lemma 2.6.** *If  $p = 3$ , then  $\dim_{\mathbb{F}_q} J(\mathbb{F}_q S_5) = 50$ .*

**Proof.** The proof is similar to that of Lemma 2.2.  $\square$

**Theorem 2.7.** *If  $p = 3$ , then*

$$\frac{\mathbb{F}_q S_5}{J(\mathbb{F}_q S_5)} \cong \mathbb{F}_q \oplus \mathbb{F}_q \oplus \mathbb{M}_4(\mathbb{F}_q) \oplus \mathbb{M}_4(\mathbb{F}_q) \oplus \mathbb{M}_6(\mathbb{F}_q).$$

**Proof.** An element  $x \in S_5$  is 3-regular if  $3 \nmid o(x)$ . We note that elements corresponding to the conjugate classes of  $e$ ,  $a$ ,  $b$ ,  $ab$ , and  $(ab)^2$  are 3-regular. As in Theorem 2.3,  $\frac{\mathbb{F}_q S_5}{J(\mathbb{F}_q S_5)}$  has dimension 70 over  $\mathbb{F}_q$  and has 5 simple components. That is,

$$\frac{\mathbb{F}_q S_5}{J(\mathbb{F}_q S_5)} \cong \mathbb{F}_q \oplus \mathbb{M}_{n_1}(\mathbb{F}_q) \oplus \mathbb{M}_{n_2}(\mathbb{F}_q) \oplus \mathbb{M}_{n_3}(\mathbb{F}_q) \oplus \mathbb{M}_{n_4}(\mathbb{F}_q). \quad (3)$$

Using dimension constraints on (3), we have

$$70 = 1 + n_1^2 + n_2^2 + n_3^2 + n_4^2. \quad (4)$$

Now observe that assignments  $a \mapsto 1, b \mapsto 1$  and  $a \mapsto 1, b \mapsto -1$  give two irreducible  $\mathbb{F}_q$ -representation of  $S_5$ . As in Theorem 2.3, using dimension of Specht modules, we see that only possible solution of (4) is 1, 4, 4, 6. Hence

$$\frac{\mathbb{F}_q S_5}{J(\mathbb{F}_q S_5)} \cong \mathbb{F}_q \oplus \mathbb{F}_q \oplus \mathbb{M}_4(\mathbb{F}_q) \oplus \mathbb{M}_4(\mathbb{F}_q) \oplus \mathbb{M}_6(\mathbb{F}_q),$$

as desired.  $\square$

**Corollary 2.8.** *If  $p = 3$ , then*

$$\mathcal{U}(\mathbb{F}_q S_5) \cong (1 + J(\mathbb{F}_q S_5)) \rtimes (\mathbb{F}_q^* \times \mathbb{F}_q^* \times GL_4(\mathbb{F}_q) \times GL_4(\mathbb{F}_q) \times GL_6(\mathbb{F}_q)),$$

where  $1 + J(\mathbb{F}_q S_5)$  is a non-abelian subgroup of  $\mathcal{U}(\mathbb{F}_q S_5)$  with order  $3^{50n}$  and  $J(\mathbb{F}_q S_5)$  is a nilpotent ideal.

**Proof.** The proof is similar to that of Corollary 2.4.  $\square$

**Theorem 2.9.** *If  $p = 2$ , then  $\dim_{\mathbb{F}_q} J(\mathbb{F}_q S_5) = 87$  and*

$$\frac{\mathbb{F}_q S_5}{J(\mathbb{F}_q S_5)} \cong \mathbb{F}_q \oplus \mathbb{M}_4(\mathbb{F}_q) \oplus \mathbb{M}_4(\mathbb{F}_q).$$

**Proof.** We know that 2-regular partitions of 5 are  $(3, 2), (4, 1)$  and  $(5)$ . The dimension of an irreducible representation of  $S_5$  over any field of characteristic 2 is given by dimensions of  $D^{(5)}, D^{(4,1)}$  and  $D^{(3,2)}$ . Now  $\dim_{\mathbb{F}_q} D^{(3,2)} = 4$  as shown in Example 5.2 in [10]. Also  $\dim M^{(5)} = 1$  and hence  $\dim_{\mathbb{F}_q} D^{(5)} = 1$ . Again  $\dim_{\mathbb{F}_q} D^{(4,1)} = \dim_{\mathbb{F}_q} \frac{S^{(4,1)}}{S^{(4,1)} \cap S^{(4,1)\perp}} = 5 - 1 = 4$ , as  $\dim S^{(n-1,1)} = n - 1$  and  $S^{(n-1,1)\perp} \cong S^{(n)}$ . The modular representations of  $S_5$  are then of dimension 1, 4, 4, which means that  $\frac{\mathbb{F}_q S_5}{J(\mathbb{F}_q S_5)}$  has dimension 33 and  $\dim_{\mathbb{F}_q} J(\mathbb{F}_q S_5) = 87$ . Also  $\frac{\mathbb{F}_q S_5}{J(\mathbb{F}_q S_5)}$  has 3 simple components. Thus  $\frac{\mathbb{F}_q S_5}{J(\mathbb{F}_q S_5)} \cong \mathbb{F}_q \oplus \mathbb{M}_{n_1}(\mathbb{F}_q) \oplus \mathbb{M}_{n_2}(\mathbb{F}_q)$ . Using dimension constraints, we have

$$33 = 1 + n_1^2 + n_2^2.$$

Only solution of this equation is  $n_1 = 4, n_2 = 4$ . Hence

$$\frac{\mathbb{F}_q S_5}{J(\mathbb{F}_q S_5)} \cong \mathbb{F}_q \oplus \mathbb{M}_4(\mathbb{F}_q) \oplus \mathbb{M}_4(\mathbb{F}_q).$$

This completes the proof.  $\square$

**Corollary 2.10.** *If  $p = 2$ , then*

$$\mathcal{U}(\mathbb{F}_q S_5) \cong (1 + J(\mathbb{F}_q S_5)) \rtimes (\mathbb{F}_q^* \times GL_4(\mathbb{F}_q) \times GL_4(\mathbb{F}_q)),$$

where  $1 + J(\mathbb{F}_q S_5)$  is a non-abelian subgroup of  $\mathcal{U}(\mathbb{F}_q S_5)$  with order  $2^{87n}$  and  $J(\mathbb{F}_q G)$  is a nilpotent ideal.

**Proof.** The proof is similar to that of Corollary 2.4.  $\square$

**Theorem 2.11.** *If  $p > 5$ , then*

$$\mathbb{F}_q S_5 \cong \mathbb{F}_q \oplus \mathbb{F}_q \oplus \mathbb{M}_4(\mathbb{F}_q) \oplus \mathbb{M}_4(\mathbb{F}_q) \oplus \mathbb{M}_5(\mathbb{F}_q) \oplus \mathbb{M}_5(\mathbb{F}_q) \oplus \mathbb{M}_6(\mathbb{F}_q).$$

**Proof.** By [20, Proposition 3.6.11], we have

$$\mathbb{F}_q S_5 = \mathbb{F}_q S_5 e_{S'_5} \oplus \mathbb{F}_q S_5 (e_{S'_5} - 1)$$

where  $e_{S'_5} = e_{A_5} = \frac{\widehat{A_5}}{|A_5|} = \frac{\sum_{\sigma \in A_5} \sigma}{60}$ . Note that  $\mathbb{F}_q S_5 e_{S'_5}$  is the sum of all commutative simple components of  $\mathbb{F}_q S_5$  and  $\mathbb{F}_q S_5 (e_{S'_5} - 1)$  is the sum of all non-commutative simple components of  $\mathbb{F}_q S_5$ . However,

$$\mathbb{F}_q S_5 e_{S'_5} \cong \mathbb{F}_q \left( \frac{S_5}{S'_5} \right) \cong \mathbb{F}_q (C_2) \cong \mathbb{F}_q \oplus \mathbb{F}_q.$$

Therefore, by Wedderburn Decomposition Theorem,

$$\mathbb{F}_q S_5 \cong \mathbb{F}_q \oplus \mathbb{F}_q \oplus \sum_{i=1}^5 \mathbb{M}_{n_i}(\mathbb{F}_{q^{k_i}}),$$

where  $n_i \geq 2$  for  $1 \leq i \leq 5$ .

Since  $p > 5$ ,  $q = p^n \equiv \pm 1 \pmod{6}$ . Also each element of  $S_5$  is  $p$ -regular. Again  $|S_{\mathbb{F}_q}(\gamma_g)| = 1$  for each  $g \in S_5$ . Hence each  $k_i = 1$  in the Wedderburn Decomposition of  $\mathbb{F}_q S_5$ .

Using dimension constraints, we have

$$\dim_{\mathbb{F}_q} \mathbb{F}_q S_5 = 1 + 1 + n_1^2 + n_2^2 + n_3^2 + n_4^2 + n_5^2$$

$$120 = 1 + 1 + n_1^2 + n_2^2 + n_3^2 + n_4^2 + n_5^2,$$

that is,

$$118 = n_1^2 + n_2^2 + n_3^2 + n_4^2 + n_5^2 \quad (5)$$

where  $n_i \geq 2$ ,  $1 \leq i \leq 5$ . But we know that dimension of Specht modules corresponding to the conjugate partition are same. Hence we have  $\dim_{\mathbb{F}_q} D^{(3,2)} = \dim_{\mathbb{F}_q} D^{(2,2,1)}$  and  $\dim_{\mathbb{F}_q} D^{(4,1)} = \dim_{\mathbb{F}_q} D^{(2,1,1,1)}$ . The only possible solution of (5) satisfying these conditions is 4, 4, 5, 5, 6. Thus

$$\mathbb{F}_q S_5 \cong \mathbb{F}_q \oplus \mathbb{F}_q \oplus \mathbb{M}_4(\mathbb{F}_q) \oplus \mathbb{M}_4(\mathbb{F}_q) \oplus \mathbb{M}_5(\mathbb{F}_q) \oplus \mathbb{M}_5(\mathbb{F}_q) \oplus \mathbb{M}_6(\mathbb{F}_q).$$

This proves the result.  $\square$

**Corollary 2.12.** *If  $p > 5$ , then*

$$\mathcal{U}(\mathbb{F}_q S_5) \cong \mathbb{F}_q^* \times \mathbb{F}_q^* \times GL_4(\mathbb{F}_q) \times GL_4(\mathbb{F}_q) \times GL_5(\mathbb{F}_q) \times GL_5(\mathbb{F}_q) \times GL_6(\mathbb{F}_q).$$

**Proof.** Proof is a simple application of the fact that for any two rings  $R_1$  and  $R_2$ ,  $\mathcal{U}(R_1 \oplus R_2) = \mathcal{U}(R_1) \times \mathcal{U}(R_2)$ .  $\square$

Combining the results from Theorem 2.3, Theorem 2.7, Theorem 2.9, and Theorem 2.11, we get the following.

**Theorem 2.13.** *Let  $\mathbb{F}_q$  be any finite field with  $q = p^n$  elements and  $S_5$  denote the symmetric group of degree 5. Then*

$$\frac{\mathbb{F}_q S_5}{J(\mathbb{F}_q S_5)} \cong \begin{cases} \mathbb{F}_q \oplus \mathbb{M}_4(\mathbb{F}_q) \oplus \mathbb{M}_4(\mathbb{F}_q), & \text{if } p = 2 \\ \mathbb{F}_q \oplus \mathbb{F}_q \oplus \mathbb{M}_4(\mathbb{F}_q) \oplus \mathbb{M}_4(\mathbb{F}_q) \oplus \mathbb{M}_6(\mathbb{F}_q), & \text{if } p = 3 \\ \mathbb{F}_q \oplus \mathbb{F}_q \oplus \mathbb{M}_3(\mathbb{F}_q) \oplus \mathbb{M}_3(\mathbb{F}_q) \oplus \mathbb{M}_5(\mathbb{F}_q) \oplus \mathbb{M}_5(\mathbb{F}_q), & \text{if } p = 5 \\ \mathbb{F}_q \oplus \mathbb{F}_q \oplus \mathbb{M}_4(\mathbb{F}_q) \oplus \mathbb{M}_4(\mathbb{F}_q) \oplus \mathbb{M}_5(\mathbb{F}_q) \oplus \mathbb{M}_5(\mathbb{F}_q) \oplus \mathbb{M}_6(\mathbb{F}_q), & \text{if } p > 5. \end{cases}$$

Finally, we remark that all calculations are verified by using GAP version 4.7.8 of 09-Jun-2015.

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